Matrix String Description of Cosmic Singularities in a Class of Time-dependent Solutions

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Abstract

A large class of time-dependent solutions with $\frac{1}{2}$ supersymmetry were found previously. These solutions involve cosmic singularities at early time. In this paper, we study if matrix string description of the singularities in these solutions with backgrounds is possible and present several examples where the solutions can be described well in the perturbative picture.

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1 Introduction

Study of time-dependent solutions in string theories is an important subject for its application to cosmology and understanding our spacetime [1]-[5]. Quite often, these solutions involve spacetime singularities. At the same time the theory becomes strong coupling and we cannot make definite statement there because perturbative picture breaks down. It is a major challenge to understand if these singularities are resolved or not, and if so how.

The solutions with partial supersymmetry (BPS solutions) may be helpful in this respect because these allow us to discuss nonperturbative regions of our spacetime. It is known that the requirement of unbroken partial supersymmetry restricts the solutions to those with null or time-like Killing spinors [6].

Recently such time-dependent solutions have been studied in the linear dilaton background in the null direction with $\frac{1}{2}$ supersymmetry and various extensions have been considered [7]-[22]. It has been argued that it is possible to map the theory to the dual matrix theory and then to matrix string theory in type IIB picture [23] where we can use S-duality to put the theory to the weak coupling region. In this way, it has been suggested that the problem of singularity in the spacetime is under control in this setting, at least for the linear dilaton backgrounds [7].

The solution studied in [7] does not involve any background in the forms which are present in the string theories. However it has not been clear if the results are restricted to only this very special solution or they are valid for other related solutions with possible backgrounds. It is thus important to study how large class of solutions allow such description of singularities in terms of matrix strings. In a previous paper [14], we have found a large class of solutions of this type in M-theory. However we have not discussed if this class of solutions have matrix string description though it is expected that this is the case. The purpose of this paper is to fill the gap and present a more detailed discussion of this problem.

This paper is organized as follows. In sect. 2, we first summarize our solutions and propose matrix string description. In sect. 3.1, starting from the discrete light-cone quantization, we show how our solutions can be mapped to the matrix string theory in the weak coupling region, and in sect. 3.2 discuss the range of validity of this approach. In sect. 4, we present several interesting examples. Sec. 5 is devoted to summary and discussions.

2 Solutions and the corresponding matrix string theories

Let us start with a short review of our solutions in 11D supergravity [14]. The metric of our background is

$$ds^2_{11} = -2e^{2u_0}dudv + \sum_{i=1}^{9} e^{2u_i}(dx^i)^2, \tag{1}$$

where all metrics are functions of u and v. We also have the four-form background

$$F = (\partial_u E du + \partial_v E dv) \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{2}$$

Upon dimensional reduction to 10 dimensions, we have

$$ds^2_{11} = e^{-\frac{2}{3}\phi}ds^2_{10} + e^{\frac{4}{3}\phi}(dx^9)^2,$$
$$ds^2_{10} = -2e^{2v_0}dudv + \sum_{i=1}^{8} e^{2v_i}(dx^i)^2, \tag{3}$$

where ϕ is a dilaton given by $\phi = \frac{3}{2}u_9$ (and $u_i = v_i - \frac{1}{3}\phi,\ i = 0, 1, \cdots, 8$).

Within this ansatz, we found that the requirement of remaining supersymmetry restricts the solutions to only u-dependent functions, and the only condition that must be satisfied is

$$\sum_{i=1}^{8} v''_i + \sum_{i=1}^{8} (v'_i)^2 - 2\phi'' + \frac{1}{2}e^{-2(v_1+v_2+v_3-\phi)}(E')^2 = 2v'_0(\sum_{i=1}^{8} v'_i - 2\phi'), \tag{4}$$

where prime denotes a derivative with respect to u, and the number of remaining supersymmetry is $\frac{1}{2}$.

Equation (4) is an ordinary differential equation for 11 functions $v_i\ (i = 0, \cdots, 8), \phi$ and E. We can regard eq. (4) as determining E for given metrics. This in fact gives a very large class of solutions in $D = 11$ supergravity, generalizing those discussed in Ref. [7].

In the simplest solution with the zero four-form, which is the linear dilaton in the null direction, the string coupling is given by

$$g_s = e^{-Qu} \tag{5}$$

for positive Q. In this string picture, there appears a singularity in the infinite past $u \to -\infty$, but we see that the string coupling diverges there. So we cannot say anything

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definite in this picture. The suggestion is that with 16 supercharges, it is possible to map the solution to matrix theory which gives us nonperturbative definition of the theory [7]. However, this is not enough since the theory is still in the strong coupling region where we do not know how to understand the behaviors of the theory. We can then make T-duality transformation to map the theory to matrix string in type IIB setting [23] where we can further use S-duality to put the theory in the weak coupling region. In this way the singularity region gets a well-defined description. Our question is whether this is true for our more general class of solutions.

The corresponding matrix string theory is expected to be described by a $(1 + 1)$-dimensional super Yang-Mills theory with 16 supercharges [23]. The action contains eight matrix-valued fields X^i corresponding to the transverse bosonic coordinates and eight spinor coordinates θ^α:

$$
\begin{aligned}
S \;=\;\; & \frac{1}{2\pi l_s^2} \int d\tau d\sigma \; \mathrm{Tr}\Big(-\frac{1}{2}(D_\mu X^i)^2 + \theta^T \gamma_\mu D^\mu \theta - g_s^2 l_s^4 \pi^2 F_{\mu\nu}^2 \\
& + \frac{1}{4\pi^2 g_s^2 l_s^4}[X^i, X^j]^2 + \frac{1}{2\pi g_s l_s^2}\theta^T \gamma_i [X^i, \theta] \Big),
\end{aligned}
\tag{6}
$$

where $\mu = \tau, \sigma$ are the flat world-sheet indices and the coupling constant is given by

$$
g_{YM} = \frac{1}{g_s l_s},
\tag{7}
$$

and the contraction rules for the indices of the matrices are determined by the metrics (3). In sect. 3.1 and examples in sect. 4, we will give the precise definition. In the light-cone gauge, the world-sheet time coordinate τ can be chosen to be proportional to the spacetime null coordinate u or X^+ since we can choose $v_0(u) = 0$ using reparametrization in u. However we will keep $v_0(u)$ in the following discussions.

3   Matrix string description

3.1   Derivation

In the discrete light-cone quantization, we make the light-like identification

$$
v \sim v + R,
\tag{8}
$$

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and focus on a sector with light-cone momentum $p^+ = \frac{2\pi N}{R}$. We define the theory with this identification as a limit of a space-like compactification

$$(v, X^1) \sim (v + R, X^1 + \epsilon R), \tag{9}$$

where we will eventually take the limit $\epsilon \to 0$. We are interested in the Lorentz transformation which further puts the identification to

$$(X^-, X^1) \sim (X^-, X^1 + \epsilon R). \tag{10}$$

The "Lorentz transformation"

$$
\begin{aligned}
u &= \epsilon X^+, \\
v &= \frac{X^-}{\epsilon} + \frac{X^1}{\epsilon} + \frac{1}{2\epsilon} \int_0^{X^+} dX^+ e^{2v_0(\epsilon X^+) - 2v_1(\epsilon X^+)}, \\
x^1 &= X^1 + \int_0^{X^+} dX^+ e^{2v_0(\epsilon X^+) - 2v_1(\epsilon X^+)}, \\
x^i &= X^i, \qquad i = 2, \cdots, 8,
\end{aligned}
\tag{11}
$$

puts the background in the form

$$
\begin{aligned}
ds_{10}^2 &= -2e^{2U_0(X^+)} dX^+ dX^- + \sum_{i=1}^{8} e^{2U_i(X^+)} (dX^i)^2, \\
\phi &= \phi(\epsilon X^+),
\end{aligned}
\tag{12}
$$

where

$$U_i(X^+) \equiv v_i(u) = v_i(\epsilon X^+), \quad i = 0, \cdots, 8, \tag{13}$$

and with the identification (10).

We note that the radius of compactification in X^1 is small, but becomes large if we make T-duality transformation. After T-duality in the X^1 direction and then S-duality, we get

$$
\begin{aligned}
ds_{10}^2 &= e^\Phi \Big(-2e^{2U_0(X^+)} dX^+ dX^- + e^{-2U_1(X^+)} (dX^1)^2 + \sum_{i=2}^{8} e^{2U_i(X^+)} (dX^i)^2 \Big), \\
\Phi &= -\phi(\epsilon X^+) + U_1(X^+) + \log r, \quad \Big(r \equiv \frac{\epsilon R}{2\pi l_s} \Big),
\end{aligned}
\tag{14}
$$

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with the identification

$$X^1 \sim X^1 + \frac{l_s}{r}. \tag{15}$$

This is now a theory of D1-branes in a background where the string coupling is weak near the big-bang.

One can repeat the analysis of the ground state of the D1-brane theory, and fluctuations around this ground state [7]. Using the bosonic part of the Dirac-Born-Infeld action

$$S_{D1} = -\frac{1}{2\pi l_s^2} \int d\tau d\sigma e^{-\Phi} \sqrt{-\det(\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + 2\pi l_s^2 F_{\alpha\beta})}, \tag{16}$$

with the metric (14), we examine classical solutions. Set $X^2, \cdots, X^8, F_{\tau\sigma}$ to zero, and write

$$X^1 = \frac{l_s}{\epsilon R}\sigma, \tag{17}$$

so that we have the periodicity $\sigma \sim \sigma + 2\pi l_s$. Assuming that $X^\pm$ depend only on τ, the action S_{D1} reduces to

$$S_{D1} = -\frac{1}{2\pi l_s^2} \int d\tau d\sigma \Big[\frac{l_s}{\epsilon R} e^{U_0(X^+) - U_1(X^+)} \sqrt{2\partial_\tau X^+ \partial_\tau X^-} \Big]. \tag{18}$$

The nontrivial field equations following from this are

$$
\begin{aligned}
0 &= \sqrt{2\partial_\tau X^+ \partial_\tau X^-}(U_0'(X^+) - U_1'(X^+))e^{U_0(X^+) - U_1(X^+)} \\
&\quad -\partial_\tau \Big(e^{U_0(X^+) - U_1(X^+)} \sqrt{\frac{\partial_\tau X^-}{2\partial_\tau X^+}} \Big), \\
0 &= \partial_\tau \Big(e^{U_0(X^+) - U_1(X^+)} \sqrt{\frac{\partial_\tau X^+}{2\partial_\tau X^-}} \Big).
\end{aligned} \tag{19}
$$

These can be satisfied if we take

$$e^{2U_0(X^+) - 2U_1(X^+)}\partial_\tau X^+ = \partial_\tau X^-. \tag{20}$$

We then consider the classical solution obtained by solving

$$e^{2U_0(X^+)}\partial_\tau X^+ = \frac{l_s}{\sqrt{2}\epsilon R}, \quad e^{2U_1(X^+)}\partial_\tau X^- = \frac{l_s}{\sqrt{2}\epsilon R}. \tag{21}$$

These determine the relation between $X^\pm$ and τ.

We choose the gauge in which X^+ and X^1 are fixed to the above classical solution in (17) and (21), and consider the fluctuation

$$X^- = X_0^- + \sqrt{2}\, y, \tag{22}$$

where X_0^- is the classical value given by (21). We find the fluctuation to the second order

$$S_{D1} = -\frac{1}{2\pi l_s^2} \int d\tau d\sigma \Big[\Big(\frac{l_s}{\epsilon R}\Big)^2 e^{-2U_1(X^+)} + \Big(\frac{l_s}{\epsilon R}\Big)\partial_\tau y + \frac{e^{2U_1(X^+)}}{2}\Big((\partial_\sigma y)^2 - (\partial_\tau y)^2\Big)$$

$$+ \sum_{i=2}^{8} \frac{e^{2U_i(X^+)}}{2}\Big((\partial_\sigma X^i)^2 - (\partial_\tau X^i)^2\Big) - 8\pi^4 l_s^4 e^{2\phi(\epsilon X^+)} F_{\tau\sigma}^2 + \cdots \Big]. \tag{23}$$

The second term can be dropped since it is a total derivative. Comparing this action and the matrix string (6), we find that the bosonic coordinates $X^i, i = 1, \cdots, 8$ in (6) precisely agree with y and $X^i, i = 2, \cdots, 8$ in (23) if we use the classical solutions given by (21) for the background. The coupling constant on $F_{\mu\nu}$ is also written in terms of the dilaton $\phi(u) = \phi(\epsilon X^+)$. Thus we conclude that the theory can be mapped to matrix string.

3.2 Regime of validity

We repeat the analysis of the range of validity of the matrix string description [7]. Consider a scalar field T with mass m. The equation of motion in our background (3) is

$$0 = \Big(2e^{-2v_0}\partial_u\partial_v - \sum_{i=1}^{8} e^{-2v_i}\partial_{x^i}\partial_{x^i} - 2(\phi' + v_0')e^{-2v_0}\partial_v + m^2\Big)T, \tag{24}$$

where prime denotes derivative with respect to u. A basis of solutions is

$$T(u,v,x^i) = \exp\Big[\phi + v_0 - i\Big\{p^+ v - \sum_{i=1}^{8} k_i x^i + \frac{1}{2p^+}\int du \Big(\sum_{i=1}^{8} e^{2v_0 - 2v_i} k_i^2 + m^2 e^{2v_0}\Big)\Big\}\Big], \tag{25}$$

in the background (3). Writing this in our new coordinate system (11), we then find that due to the identification (10), its momentum is quantized:

$$p^+ = \epsilon k_1 - \frac{2\pi n}{R}. \tag{26}$$

We consider only $n = 0$ case, and set

$$p^+ = \epsilon k_1. \tag{27}$$

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Requiring its mass squared is positive, we find that

$$|k_1| \leq \frac{2u}{\int du e^{2v_0(u) - 2v_1(u)}} \epsilon E^-, \tag{28}$$

where E^- is the light-cone energy. Similarly

$$|k_i| \sim \frac{u}{\left| \int du e^{2v_0(u) - 2v_1(u)} \int du e^{2v_0(u) - 2v_i(u)} \right|^{\frac{1}{2}}} \epsilon E^-. \tag{29}$$

Examining other components, we find that the momenta in various directions are of order

$$
\begin{aligned}
X^+ &: \quad \epsilon E^-, \\
X^- &: \quad \epsilon E^- \times \frac{u}{\int du e^{2v_0(u) - 2v_1(u)}}, \\
X^i &: \quad \epsilon E^- \times \frac{u}{\left| \int du e^{2v_0(u) - 2v_1(u)} \int du e^{2v_0(u) - 2v_i(u)} \right|^{\frac{1}{2}}}.
\end{aligned} \tag{30}
$$

From the metric (14), the effective time-dependent string length l_s^{eff} in the 10-dimensional spacetime is read as

$$l_s^{\text{eff}} = l_s e^{\frac{\phi(\epsilon X^+)}{2} - \frac{v_1(\epsilon X^+)}{2}} \sqrt{\frac{2\pi l_s}{\epsilon R}}. \tag{31}$$

The condition for massive open string to decouple is given by

$$E_o l_s^{\text{eff}} \ll 1, \tag{32}$$

where E_o is typical energy appearing in this theory, which are the energy scales given in (30). This should be satisfied in the $\epsilon \to 0$ limit. We will check that in all examples given below, these can be satisfied.

The condition for gravity to decouple is as follows: The ten-dimensional effective Newton constant G_N^{eff} is

$$G_N^{\text{eff}} \sim \frac{4\pi^2 l_s^{10}}{\epsilon^2 R^2} e^{2\phi(\epsilon X^+) - 2v_1(\epsilon X^+)}, \tag{33}$$

and the strength of the gravity is

$$G_N^{\text{eff}} E_o^8 \sim \frac{4\pi^2 l_s^{10}}{\epsilon^2 R^2} e^{2\phi(\epsilon X^+) - 2v_1(\epsilon X^+)} E_o^8, \tag{34}$$

which should vanish in the limit $\epsilon \to 0$. We will show that this is also valid in all the examples below. As long as the conditions (32) and (34) are satisfied, we may have well-defined matrix string description.

4 Examples

Let us now discuss some simple but interesting cases.

4.1 Linear solutions

As a simple example, we consider

$$v_i(u) = a_i u, \quad (i = 0, 1, \cdots, 8), \quad \phi(u) = a_9 u. \tag{35}$$

We choose $a_9 < 0$ to have singularity at $u \to -\infty$ and the theory is strong coupling there. This can be always done by the choice of u. This includes the linear dilaton considered in Ref. [7]. From (21), our classical solutions are

$$\epsilon X^+ = \begin{cases} \frac{1}{2a_0} \log\left(\frac{\sqrt{2}l_s}{R} a_0 \tau\right), & (a_0 \neq 0) \\ \frac{l_s}{\sqrt{2}R} \tau, & (a_0 = 0) \end{cases}, \tag{36}$$

and we have

$$U_i(X^+) = a_i \epsilon X^+, \quad \phi(\epsilon X^+) = a_9 \epsilon X^+. \tag{37}$$

It is then easy to check that all the conditions (32) and (34) are satisfied. The only remaining constraint is the Einstein equation (4), which simply determines E'.

4.2 Logarithmic solutions

As a slight generalization of the simple linear background, we can consider

$$v_i(u) = a_i \log|u|, (i = 0, 1, \cdots, 8), \quad \phi(u) = a_9 \log|u|. \tag{38}$$

We choose $a_9 < 0$, and the string coupling becomes large near $u \sim 0$. Here we have more nontrivial background E', and this case includes spacetime similar to that in Ref. [1]. Eq. (21) gives

$$\epsilon X^+ = \left[\frac{l_s}{\sqrt{2}R}(2a_0 + 1)\tau\right]^{\frac{1}{2a_0+1}}, \tag{39}$$

and

$$U_i(X^+) = a_i \log(\epsilon X^+), \quad \phi(\epsilon X^X) = a_9 \log(\epsilon X^+), \tag{40}$$

both of which are well-defined in the $\epsilon \to 0$ limit. It can be easily checked that the conditions (32) and (34) give

$$
\begin{aligned}
1 + a_9 - a_1 &> 0, \\
1 + a_9 + a_1 + 2a_i - 4a_0 &> 0, \\
3 + a_9 + 3a_1 + 4a_i - 8a_0 &> 0,
\end{aligned}
\tag{41}
$$

for all $i = 1, \cdots, 8$. When these are satisfied, we can get the matrix string description.

4.3   Linear and logarithmic solutions

As a more nontrivial example, let us consider

$$
v_i(u) = a_i u, \quad (i = 0, \cdots, 9 - d), \quad v_j(u) = a_j \log u, \quad (j = 10 - d, \cdots, 9), \quad \phi(u) = a_9 u. \tag{42}
$$

In this case, eq. (21) gives again (36) and

$$
U_i(X^+) = a_i \epsilon X^+, \quad U_j(X^+) = a_j \log\left(\epsilon X^+\right), \quad \phi(f) = a_9 \epsilon X^+, \tag{43}
$$

are all well-defined in the $\epsilon \to 0$ limit.

The conditions (32) and (34) give

$$
a_j > -1. \tag{44}
$$

5   Summary and discussion

It is a very interesting and important problem to examine if and how string theory can resolve the singularities of our spacetime. In this paper we have discussed one possible direction in the context of matrix string theory. Starting from the general class of time-dependent solutions with $\frac{1}{2}$ supersymmetry found in [14], we have formulated how the solutions can be mapped to weak coupling matrix string theory, where we can examine the behavior of the theory using perturbative picture.

It appears that we can always map the theory for the $\frac{1}{2}$ BPS solutions to matrix string description in the presence of backgrounds, and there is certain range of validity in this approach. We have also given several examples in our solutions. Although we have given

several examples and there are range of validity on the parameters in the solutions, we can expect that there is always valid range for the matrix string description.

Even though there are singularities in the geometrical string picture, the mapped matrix string theories do not show any violent behavior. However geometric picture how our spacetime behaves there is not clear. Our results are consistent with the recent suggestion that our spacetime is an emergent one. It would be interesting to further explore the physical picture as to what happens close to the singularities, and also study other cases where cosmic singularities are tamed.

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